Neuropsychology and the Instructional Psychology of Mathematics

Vincent J. Glennon
The University of Connecticut

The First Annual
Dr. John W. Wilson Memorial Address

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NEUROPSYCHOLOGY AND THE INSTRUCTIONAL PSYCHOLOGY OF MATHEMATICS

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DEDICATION

John Wilson grew up and was educated in the public schools in Buffalo, New York. He married Nancy McClellan and they became the proud parents of Debi, Mark, John and Wendy. He served in the Fifth Army Division in Europe in 1955, before becoming a kindergarten, and later, a second grade teacher on the Onadauga Indian Reservation in New York State.

John's educational background includes a liberal arts degree at Bowling Green State University and a master's degree in epistimology at Harvard, as well as completing the Ph. D. in Education at Syracuse in 1964.

In the early 1960s he directed the country's first arithmetic clinic begun by Dr. Vincent Glennon at Syracuse for the diagnostic study and treatment of learning problems in mathematics. When Dr. Wilson moved from Syracuse in 1969, he continued to develop diagnostic courses and clinical programs for advanced degree students at the University of South Florida, Tampa, and the University of Maryland, College Park.

He is recognized as one of the original "founding fathers" of the Research Council for Diagnostic and Prescriptive Mathematics. He truly was a pioneer leader of the diagnostic mathematics movement during the past two decades. His ideas and ideals continue to live and grow through his students, his colleagues, and his writings.

At his death, John Wilson held the positions of Director of the Arithmetic Clinic, and Professor of Mathematics Education at the University of Maryland. This address is part of a series of lectures given annually at the international meeting of the RCDPM and is dedicated to the memory of John Warner Wilson.

Don Bernard

JOHN WARNER WILSON

1929 - 1979

"WE DO NOT LEARN BY DOING!
WE LEARN BY THINKING ABOUT WHAT WE ARE DOING!"

Introduction

Robert B. Ashlock

The John W. Wilson Memorial Lecture is an annual lecture established by The Research Council for Diagnostic and Prescriptive Mathematics Executive Committee as a result of a memorial fund instituted by Dr. Wilson's family. John Wilson died while attending our conference in Tampa in April 1979. Dr. Wilson was a founding member of the Research Council for Diagnostic and Prescriptive Mathematics and the concerns of council members were his concerns. He sought to find more adequate ways to diagnose a child's learning of mathematics, to prescribe corrective or remedial instruction and to implement such plans. He was concerned that our diagnosis, our prescription and our instruction of a child be done thoughtfully. In fact, John would sometimes praise his students by saying "We do not learn by doing!" Then he would add, "We learn by thinking about what we are doing!"

John was devoted to his students and believed each of them capable of becoming thinking individuals. He sincerely believed that teachers do operate from a base of theory and he sought to help them articulate their own ideas and then to rethink them. His own mind was a catalyst for new ideas that would soon be recast because of some imperfection, and recast again. Ideas were important to John, and he loved to try out his own evolving ideas on anyone who would listen, as some of you know. He had a singleness of purpose that seems to some to be inconsistent with his own tolerance of pluralities of thought in others, and yet he was a very warm person who was easy to get to know. He thoroughly enjoyed so much of life. He was full of good humor and often added a needed perspective. He enjoyed heated debate with a colleague or student, but he did not reject the individual personally. John Wilson became close friends with his colleagues and his own teachers and students, continuing his contacts with them over the decades. How appropriate that in his memory we launch a series of annual lectures designed to stimulate our own thinking. May they help each of us become more thoughtful researchers and teachers.

It is also fitting that the first John W. Wilson Memorial Lecture be given by Dr. Vincent Glennon. For Dr. Glennon was John Wilson's own teacher and advisor. Some of us will never forget John's memorable introduction of Dr. Glennon at the Fourth Annual RCDPM Conference held in Maryland. It was an introduction which I will not attempt to duplicate, but I do want to tell you just a few things about our speaker. Vincent Lennon is a professor of Mathematics Education at the University of Connecticut and was formerly the founder and director of the Arithmetic Studies Center at Syracuse. In 1959, he established an Arithmetic Clinic at Syracuse University, a clinic which John Wilson himself directed and which gave inspiration to the establishment of several other clinics across the nation. Dr. Glennon is well known for his service and leadership in the National Council of Teachers of Mathematics, in ASCD and in other state and national organizations, and he frequently speaks at regional and national conferences. His list of publications is extensive. Of particular interest to us, I think, is the new NCTM series of Basic Reference Books to be launched soon. Dr. Glennon is the editor, and author of two chapters in the first of these, a professional reference book entitled *Mathematical Education of Exceptional Children and Youth: An Interdisciplinary Approach*. The hope is to have this volume out by about November 1980.

Now some of us in this room may be known for our circles. That is, people think of us as round-faced, or we wear glasses with circular lenses and the like. Others of us in this room may be thought of as squares, but as those of you who have heard Dr. Glennon or have read his publications know our speaker this afternoon is known for his triangles. And his triangles seem to increase as the number of years go by. Dr. Glennon, we are delighted that you're with us to present the first John W. Wilson Memorial Lecture.

Foreward

I consider it a signal honor to have been invited to prepare this the first in a series of annual lectures to honor the memory of Dr. John W. Wilson. During my long association with John at Syracuse University I came to know him well. He enjoyed telling the story, often, of how we first met. It was quite by accident and in retrospect very fortuitous for me. Due to a mix-up in the office responsible for administering the master's comprehensive examination, John, not one of my advisees, was given one of my examinations in mathematics education. When it arrived on my desk for evaluation I was more than a little amazed that a candidate for the degree in general elementary education who had taken no courses specifically in mathematics education could do so very well! Needless to say, I became very much interested in meeting this person. It didn't take long! Those of you who have had the good fortune of knowing John are well aware of the fact that he was not shy and would not let an error of that order of magnitude go unchallenged. Shortly, he showed up in my office and we had a very long talk which ranged far beyond the issue at hand. Like all of his later colleagues I was greatly impressed by his verbal ability, by his earlier sound education in philosophy and logic at Bowling Green State University as an undergraduate and then at Harvard as a graduate student under such great scholar teachers as Willard Quine, and by his ability to integrate several disciplines to illuminate a point or to bolster his side of the discussion of almost any professional question.

This was the beginning of a long and very rewarding relationship between ourselves and our families. In due time John resigned from his position as a teacher at the Onondaga Indian Reservation School to accept a graduate assistantship in mathematics education. Upon the awarding of the Ph.D. degree, he was invited to join the faculty in mathematics education with specific responsibility for the courses in diagnostic-prescriptive teaching and for further developing the Arithmetic Clinic, both of which he did very well. After we left Syracuse University in 1969, we continued to write together, meet regularly at conventions and to maintain a continuing relationship between ourselves and our families.

The hallmarks of John's life, as I saw them, were his boundless enthusiasm for ideas, whether those of empirical research or rational philosophy, his enthusiasm for talking compellingly on any question of today, or yesterday, or tomorrow, whether it be political, social, theological, economic, or whatever, his passion for life and his profession, his devotion to his family and his robust friendships with his colleagues and students.

John was the kind of person of whom Robert Frost, our mutually favorite poet, wrote:

A Time to Talk

When a friend calls to me from the road
And slows his horse to a meaning walk,
I don't stand still and look around
On all the hills I haven't hoed,
And shout from where I am, "What is it?"
No, not as there is time to talk.
I thrust my hoe in the mellow ground,
Blade-end up and five feet tall.
And plod: I go up to the stone wall
For a friendly visit.

Neuropsychology and the Instructional Psychology of Mathematics

The title of this address, "Neuropsychology and the Instructional Psychology of Mathematics," suggests an effort which cannot be realized in any depth in a luncheon-length address. Perhaps it cannot be attained well in a fifteen-week seminar, or even a degree-length program of study. However, it is an issue of the first order of magnitude for us today. This audience and this organization, the Research Council for Diagnostic and Presctiptive Mathematics is, in my opinion, the singular professional group which is small enough and cohesive enough to take the leadership in the development of a sound instructional psychology. We need this as a substrate of the mathematic programm for *all* children -- the socially and emotionally impaired, the talented and gifted, the slow learners and the metally retarded, the blind and hearing-impaired, those with other major physical impairments and, that large group of children, youth and adults who are not defined as exceptional in any educationally significant way -- the "average" ones. This audience and this organization has the responsibility for going beyond the limited metes and bounds of mathematics education as they are commonly evidenced. It is not enough to view the instructional psychology of mathematics simply as a bag of methods and materials applied to the computational skills to be selected at will much as the plumber, the carpenter, or the auto mechanic selects from his tool kit to stop a leak, to repair a door, or to replace a carburetor.

As the social philosopher-psychologist John W. Gardner once said, and I paraphrase, Society must value its teachers at least as much as it does its plumbers. For if it does not, in time neither its theories nor its pipes will hold water. And we teachers must value and advance our emerging theory of an instructional psychology just as the unusually skillful diagnostic physician guards his theory and practice of medicine. The truly professional physician is constantly aware of any developments and breakthroughs in the disciplines which are the substrates of the interdiscipline of medicine. By way of an illustration, the truly skillful physician of today is continuously aware of progress in the newly developing areas of behavioral medicine and orthomolecular medicine. That is, he views the theory and practice of behavioral medicine as an interdiscipline which integrates the principles of medicine, physiology, psychiatry, and psychology for the better diagnosis and treatment of the patient.

In the same way, we need to continually keep abreast of newly developing disciplines which relate to and enhance the instructional psychology of mathematics. Most significant among these is the development in the last four years of noninvasive procedures for studying the cognitive functions of each hemisphere of the brain and the role of their connecting tissue, the *corpus callosum*. This development adds a new dimension to our view of mathematics education as an interdiscipline. And the disciplines between the disciplines are the most important disciplines for improving the human condition.

As Marcel Kinsbourne (1978) recently said, "Few topics in the neurosciences can match the study of cerebral lateralization in its power to stimulate the imagination of people." So, it behooves this organization to keep in close touch with the research being done in neuropsychology in order to select out of the findings those which are of use to us in our college and university courses in diagnostic and prescriptive teaching and in our clinical work with children, youth, and adults.

Our Short History

The instructional psychology of mathematics has a long past but a short history. The academic roots of our profession in the distant past can be found in the teaching style of Socrates. In his dialogue *Meno* Plato illustrates how Socrates' teaching style was based upon his (mis)understanding that infants were born with all knowledge.In the light of our knowledge of the neuropsychology of today that would be comparable to saying that neonates are born with a brain weighing approximately three pounds (rather than three-quarters of one pound) and that brain is fully myelinated at birth, and has all knowledge engrams stored for ready retrieval at will. Receiving through the senses as a function of learning would not be a concern of that view of instructional psychology for the obvious reason that all knowledge would be in place at birth. This erroneous perception of the neurological function of the learning process determined Socrates' style of teaching. That is, to Socrates teaching was the act of causing someone to reminisce and thereby to bring the already stored knowledge to the expressive level of speaking, writing, gesturing, etc. Such a view of the functioning of the brain is not valid in today's neuropsychology and is therefore not a part of a sound instructional psychology of mathematics.

Our *short history* begins with the faculty psychology which dominated the instructional psychology of the latter half of the nineteenth century. Built upon the work of the eighteenth century philosopher and mathematician Christian Wolff, the mind was presumed to consist of several faculties or capacities: imagination, memory, perception, reasoning, the will, and others. It was further believed that each of these faculties could be strengthened through mental discipline in much the same way that a set of muscles can be strengthened through physical discipline or practice. Therefore the good school mathematics program was one that provided exercises, both mental and written, which were difficult even distasteful. Like the cold and cough medicines of the day, the more difficult the mathematics was to "swallow" the more good it was doing.

As we all know, the scientific movement in psychology was in its infancy at the end of the nineteenth century and the beginning of this century. And it was the application of the scientific method to the study of the question of mental discipline which brought about its end as a significant theoretical basis for an instructional psychology. The research of Edward Lee Thorndike and Robert S. Woodworth (1901) which was reported in 1901 provided the *coup de grace.* It was necessary then, to seek a more viable basis for an instructional psychology of mathematics.

Thorndike (1922) himself provided that basis through the application of a stimulus-response model of learning resting upon his three laws of learning: readiness, exercise, and effect. Through his instructional psychology which placed great centrality on the number of practices necessary for the facilitation of a given (neural) bond, Thorndike unwittingly gave the classroom teacher a "license" for the use of drill in inappropriate and miseducative ways.

But listen to Thorndike's own words:

> For one of the easier (neural) bonds, most facilitated by other (neural) bonds (such as 2x5 = 10, or 10-2 = 8, or the double bond 7 = two 3's and 1 remainder) in the case of the median or average pupil, twelve practices in the first week of learning, supported by twenty-five during the two months following, and maintained by thirty practices well spread over the later periods should be enough. For the more gifted pupils lesser amounts down to six, twelve, and fifteen may suffice. For the less gifted pupils more may be required up to thirty, fifty, and a hundred. It is to be doubted, however, whether pupils requiring nearly two hundred repetitions of each of these easy bonds should be taught arithmetic beyond a few matters of practical necessity. (p. 133)

These estimates may be reasonable. But experienced teachers gave an even higher importance to the number of practices needed. This is shown in their estimates of the content of the two books used in those years to cover the work of the first six grades. When asked by Thorndike to estimate the number of practices experienced over the six year period for the subtraction fact 5-3, their lowest estimate was 20; their median estimate was 1,100; and their highest estimate was 2,500,000! (S/he should live so long!)

Thorndikian S-R, scientific based, instructional psychology as it was implemented in the classroom was narrow indeed.

The roots of our present and broader instructional psychology were formed by several people during the first half of this century and are exemplified in their varied contributions. Guy M. Wilson (1939) and his colleagues gave much visibility to a theory of curriculum for selecting the mathematics that is of most worth for teaching. To Wilson, the school program should include only the mathematics (arithmetic) that was clearly of use in everyday situations in adult life.

Charles H. Judd (1936), perhaps our first true instructional psychologist or cognitive psychologist, emphasized the importance of education as the process by which we cultivate the higher mental processes. He defined a higher mental process as "one to which the individual makes a large contribution through his own conscious effort." (p. 39)

Judd contrasted the prevailing drill method of teaching with the higher mental processes approach:

> The chief difficulties encountered by present-day teaching of arithmetic arise from the fact that schools, at least the better schools, are attempting to develop in pupils an understanding of number rather than merely drilling them in the use of tables and formal rules. When the effort is made to develop understanding, education is aiming at the cultivation of higher mental processes. To cultivate higher mental processes is infinitely more difficult than to teach methods of solving problems by formulas held in memory and applied mechanically. (p. 41-42)

Guy T. Buswell and Leo J. Brueckner and others contributed significantly through their atomistic analyses of the common difficulties in processing the standard algorithms.

The fountain head of sound instructional psychology was the Lincoln School of Teachers College, Columbia University, begun in 1917. Among this "unusually imaginative faculty", as Lawrence Cremin (1961) described it, was John R. Clark, our Honorary President of the National Council of Teachers of Mathematics. The staff proceeded to produce a whole new array of materials to implement the new instructional psychology of the Progressive Education Movement: "curriculum guides, textbooks, workbooks, teaching units, and achievement tests. Most important, the staff ran a first-rate school...and a pioneering spirit pervaded the activities of teachers, students, and parents alike." (p. 282). Personally, and through his graduate students, Clark made a profound influence on the mathematics programs of the nation's schools.

The most influential instructional psychologist in our short history who specialized in the teaching and learning of mathematics was William A. Brownell, one of Judd's students. Through his research and cogent essays, Brownell, quite single-handedly, shifted the basis for the mathematics education program from the "drill theory", which dominated the instructional program, to the "meaning theory." Brownell (1935) defined the meaning theory as one which "conceives of arithmetic as a closely knit system of understandable ideas, principles, and processes. According to this theory, the test of learning is not mere mechanical facility in "figuring." The true test is an intelligent grasp upon number relations and the ability to deal with arithmetical situations with proper comprehension of their mathematical as well as their practical significance." (p. 19)

During these same years, the genetic epistemologist and cognitive psychologist Jean Piaget provided a wholly new and stimulating dimension to the instructional psychology of mathematics. In his pursuit of a logical link between the biology and the psychology of human development, Piaget and his associates, over a period of a half century, created ingenious research techniques. Many of the tasks which were created to elicit the child's behavior had a mathematical basis. Because of this, we in the instructional psychology of mathematics, are among the primary beneficiaries of this theory of cognitive development. It is up to us to determine the logical implications of his findings for our diagnostic and prescriptive teaching of mathematics in particular, and for our instructional psychology in general.

A New Dimension

During the many centuries in our *long past* of instruction in mathematics, our efforts to improve the processes of selecting, transmitting and evaluating that part of the culture have been rooted in rational psychology. Whether our efforts were successful or not we do not really know. In our *short history* which began with the rapid growth of scientific psychology, research methods of the scholars named above and others were limited to the measurement and evaluation of outward observable behaviors. These behaviors were reflections of inner neuropsychological mathematical functions called engrams. An engram is a neuropsychological representation of a stimulus.

Some fifty years ago Samuel Orton provided a new dimension for the study of brain behavior. Of course it had been known for centuries that a cerebral insult in the form of damage from the outside, or blood clot, or stroke or disease from the inside, could directly affect a person's cognitive performance with either or both of our symbol systems, the alphabetic and the numeric (as well as psychomotor and affective behavior). The result could be the loss of speech, loss of ability to compute, etc. But our short history assigns to Orton (1937) the distinction for providing a rationale for language disabilities. And here, it is important that we keep in mind that reading is language-dependent. Hence, the reading of mathematics is language-dependent. The syntax of the sentence, "Tom and Bill are friends" is the same as the syntax of the number sentence "Three and (plus) five are (is) eight."

Orton was a neurologist with particular interests in neuropathology and neuropsychology. He proposed the idea that developmental language disabilities (as opposed to disabilities that are losses due to cerebral insults) were due to incomplete development of functional superiority in the dominant hemisphere. In the half century since then, and with a rapid rate of acceleration in the past five to ten years, the study of hemisphere function through non-invasive techniques has contributed a sizeable corpus of knowledge. Some of this knowledge has direct bearing on the processes associated with the instructional psychology of mathematics. I would like to summarize some ideas in this new dimension of diagnostc and prescriptive teaching and suggest some logical implications for our work.

At this point, I should emphasize strongly the *caveat* that we who work in the instructional psychology of mathematics, which as mentioned above is an interdiscipline, must not presume to be neurologists. In the same way, neurologists who know neither the mathematics nor the instructional psychology of mathematics should not presume to be competent in diagnosing and prescribing in the teaching and learning of mathematics. But we need each other.

Having stated my *caveat*, I can now suggest some of the cognitive processes for which there seems to be evidence for hemisphere specialization.

In general, the left hemisphere processes verbal and analytic information. In general, the right hemisphere processes visuospatial and Gestalt (holistic) information. A summary of the findings from many research studies suggests the hemispheres perform these functions:

Hemispheric Functions of the Brain

Left hemisphere functions	**Right hemisphere functions**
Verbal	Visuospatial (including gestural communication)
Logical	Analogical, intuitive
Analytic	Synthetic
Linear	Gestalt, holistic
Sequential	Simultaneous and multiple processing
Conceptual similarity	Structural similarity

Therefore, in general, for almost all (98%) right-handed people, and for about two-thirds or so of all left-handed people, arithmetic, language, reading and similar cognitive activities are primarily left-hemisphere functions. And, in general, primitive musical activities, visual arts, architecture, sculpturing, etc. are primarily right-hemisphere functions.

However, a classification and listing such as the above can be an oversimplification and lead us to possibly erroneous educational implications. Out of such faulty reasoning we are liable to develop questionable principles and practices in our work in diagnostic and prescriptive teaching of mathematics. Wittrock (1978) expressed his caution this way:

> ... it should be remembered that cognitive functions cannot be reduced to neural structures, and psychological processes and educational methods should not be grafted onto the neurosciences. But the models and hypotheses developed to explain phenomena in learning, memory, attention, cognitive style, instruction and teaching can be improved by relating them to other contexts, such as neurological models of the cognitive processes of the brain. For example, psychological research on imagery and verbal processes and cognitive styles ... can be juxtaposed with recent research on the hemispheric processes of the brain. (p. 63)

But we are not without some suggestions for an instructional psychology that can be gleaned from research in recent years on the cognitive functions of the brain. One of our primary tasks, as we work with a child, is to obtain some index of the limits of his educability.To try to teach more mathematics and at a faster rate than he is ready to learn or able to learn is to frustrate him and misuse our time. A conventional procedure for assessing a person's ability to do school-type learning (commonly called "intelligence") is the use of psychometric instrument(s). In the past, there has been little concern for the possibility that such instruments might be discriminating from the viewpoint of hemispheric functioning. However, in very recent years and with our new knowledge on brain hemisphere specialization, the validity of these instruments, already questionable for other reasons, comes under even greater scrutiny.

While recognizing that much rapid communication takes place via the bundles of transverse fibers which connect the two hemispheres, the *corpus callosum*, and that it is the interrelationships between the two hemispheres which determine much of how we write, read, perform an algorithm, or ride a bicycle, it is the specialization of function of each half which controls most cognitive performance. The commonly used standardized tests of intelligence tap some, but by no means all, of the different types of mental processes of each hemisphere.

Kaufman (1979) contends that the heavy emphasis on left-brain functions which appear in the Wechsler and Binet instruments discriminates against many children, youth, and adults who are primarily "right-brain learners". Psychometricians and others who develop such tests have largely failed to recognize the phenomenon of hemispheric generalization in their selection and validation of test items. Hence, these instruments have failed to provide us with a valid measure of *global* intelligence, or interhemispheric integration. Kaufman writes:

> In particular, there is reason to believe that better assessment of right brain functions may be of special value in the measurement of the black individuals' intellectual ability. Weems stressed the importance of music and movement (both right hemisphere functions) within the black community, and suggested that the mathematical essence of music and dance may be effectively used to facilitate the teaching of arithmetic to black children. (p.103)

Another of our concerns in the instructional psychology of mathematics is that of the significance of the central processes, or underlying perceptual and cognitive processing skills, that are the presumed substrates for success in school-type learning.

The central cognitive processes include auditory discrimination and memory, visual discrimination and spatial relationships, intersensory integration, auditory sequential memory, attention disturbance and attention span, figure-ground pathology, and dissociation. The question of concern to people who are interested in finding better ways to improve diagnostic and prescriptive procedures is -- Does specific training in one or more of the central processes result in improved learning of mathematics, or reading, etc?

By way of a specific illustration, let us assume that we are working with a child who has had unusual difficulty in learning to count rationally using the set of natural numbers. The school psychologist or special educator, using their widely-accepted central processing deficit model, would identify the child's problem as an auditory-sequential memory deficit. His prescription would be that of having the child memorize and recite in sequence lists of spoken names which are arbitrarily selected -- such as color names, names of things in the classroom, names of children, etc. Presumably, this practice will result in an amelioration of the deficit and this in turn would result in a facility in counting. The mathematics educator, on the other hand, would see the child's problem as one which can be treated by improving the simpler constituent skills which together lead up to the complex skill of counting. These constituent skills would include: the one-to-one correspondence idea; the number names have a fixed sequence; the last number named tells the cardinal number of the set; and the counting idea can be applied to any set of objects (or no objects).

In a recent summary of the effectiveness of the central processing deficit model (the underlying skills model), Arter and Johnson (1979) conclude that the model does not work, that "teachers are adhering to an unvalidated model", and that it is imperative to call for a moratorium on that diagnostic-prescriptive procedure and on the training of teachers to use it.

To me, this suggests that the neuropsychology of our instructional psychology of mathematics in which we advocate *direct* diagnosis and prescription of the mathematics itself, rather than the underlying cognitive processes, is on the right road. And this in turn takes me back to the point I made earlier -- that this organization RCDPM, is the unique group that has the competencies in both the mathematics itself and the instructional psychology of mathematics to provide the leadership needed today. It is not enough to know only the general characteristics of the children and youth that we are working with and be able to identify them.

Theories of Curriculum

What then, are the important variables in an instructional psychology of mathematics (or of subject matter learning in general) and how does our new knowledge in neuropsychology contribute to them? Time limitation permits me to give only a few illustrations. (These have been discussed elsewhere: Glennon, 1980a; and Glennon, 1980b.)

Certainly one of the most significant variables in an instructional psychology is a holistic understanding, a Gestalt, of the several sources of the curriculum. How do we determine what mathematics is of most worth for teaching to a normal child? to a blind child? to a mathematically talented child? to a dyscalculic child? to a physically impaired child? to a hearing impaired child? to a socially and emotionally impaired child? to a slow learner? to an educable mentally retarded child? Surely, not all mathematics is of equal worth for each of these children, or youths, or adults. And surely no published mathematics program, despite the self-serving claims we now see in advertising brochures, can honestly purport to serve the unique needs of, say, the slow learners, the learning disabled, the economically and culturally different, and the socially and emotionally impaired.

I find it useful to use a triangular shaped model to represent the three, and there are only three, major or primary sources of the curriculum, whether it be the mathematics curriculum, the social studies curriculum, or the curriculum in general.

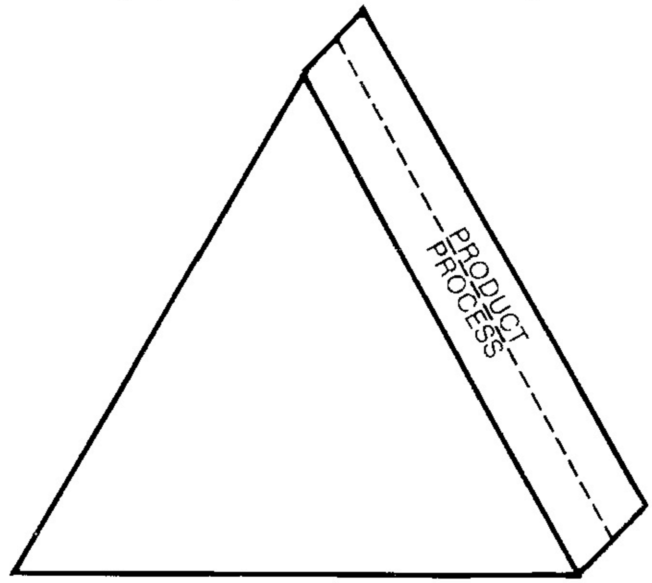

Figure 1. Model of Theories of Curriculum

In the past, and too often today, the diagnosis and prescription has been limited to the computational skills, and to the product-type outcomes of those skills. This is due in large part to the fact that commercially published diagnostic materials are primarily concerned with these outcomes. Hence they fail teachers in two ways: they intuitively suggest that the outcomes measured are the only important outcomes; and they fail to measure other important outcomes.

In the model above, I have indicated the three major sources of answering the question: what mathematics is of most worth for average children and for each type of exceptional child? Each theory has its rationale, its basic assumptions, its advantages, and its limitations.

How does our new knowledge in neuropsychology relate to this curriculum variable (theories of curriculum) in an instructional psychology? One instance need suffice. Witelson (1976) reported a study in which, using a new non-invasive test, she compared specialization of the right hemisphere for spatial processing of boys and girls between 6 and 13 years of age. It was found that, in boys, the right hemisphere has the dominant role in processing nonlinguistic spatial information as early as 6 years of age. By contrast, in girls, the right hemisphere is not dominant even by 13 years of age; rather they maintain bilateral representation. The fact that the female right hemisphere remains plastic longer (i.e. does not lateralize early in life) may account in part for their lower incidence of developmental disorders -- including dyscalculia, which is also called numerical dyslexia. And the fact that boys lateralize early may account at least in part for their high incidence of dyscalculia. About 15% of all children, youth, and adults are dyscalculic. About 90% of all dyscalculics are males.

Whereas we are quite familiar with the product-type and process-type outcomes indicated above in our curriculum model, we may not be including as part of his curriculum the kind of experiences which develop the child's ability to think critically about his work in mathematics; that is, experiences in *metacognition* (Flavell, 1979). If a third grade child is asked if he knows his addition and subtraction number stories, he might say "yes". But when he is asked to respond to them, he might evidence considerably less than the level of meaningful habituation. Although he may be able to think mathematics, he may not be able to think about his thinking. Research in this new area of an instructional psychology indicates that children, youth, and adults do relatively little in critically monitoring and appraising their cognitive processes: attention, memory, transfer, problem solving and others. Metacognition adds a new dimension to our curriculum and hence to our diagnostic-prescriptive work.

Theories of Teacher Methodology

A second important variable in an instructional psychology of mathematics is that of the teacher's methodology. Again, I find the use of a triangular shape useful to model the metes and bounds of methodology.

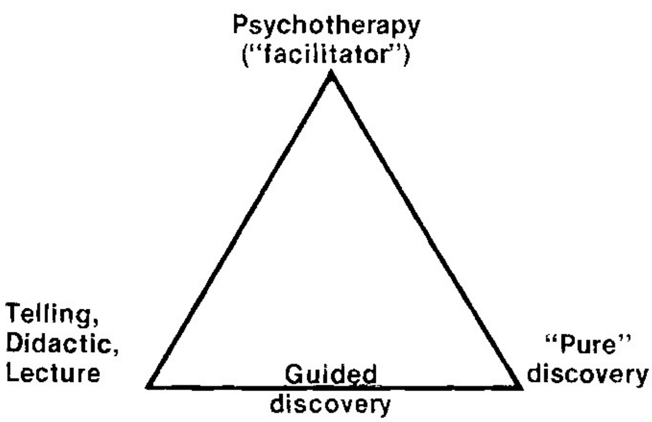

Figure 2. Model of Theories of Methodology

What does research in neuropsychology contribute to methodology? Quite a bit! And again, an instance will suffice to illustrate. A large number of studies of electroencephalographic data has been done for the purpose of comparing asymmetries in brain wave patterns when verbal or spatial tasks are being performed. A ratio between the activity in each of the hemispheres is called the alpha power. Alpha power is an index of the "idling" or inactivity of a hemisphere. The greater the alpha production in the right hemisphere the greater is the activity in the left hemisphere, and vice versa. When verbal and arithmetic tasks are being processed the alpha production in the right hemisphere is greater than in the left hemisphere; that is, the right hemisphere is "idling".

And, when the tasks are non-verbal, spatial in nature,the alpha production in the left hemisphere is greater, and hence the right hemisphere is lateralized for these visuospatial experiences. These findings suggest that the methodology of teaching must provide more learning experiences, (i.e. more stimulus nutrition) for the neglected right hemisphere than is true at the present time.Today's classroom is essentially a left hemisphere classroom. Too, our diagnostic and prescriptive procedures must be broadened. This means going considerably beyond the present standardized, largely pencil-and-paper, verbal tasks if we wish to capitalize the new knowledge in neuropsychology for a better teacher methodology and a more valid diagnostic-prescriptive program.

Theories of Motivation

A third significant variable in a well-formed instructional psychology is concerned with the source(s) of energy for the learning process – that is, theories of motivation. Motivation is characterized by the arousal and focusing of attention on the environment or an idea.

Sears and Hilgard (1964) identified three major sources of motivation: the social motives, the ego-integrative motives, and the cognitive motives. The achieving child, youth, or adult can be motivated by any one source or some combination of the three sources of energy.

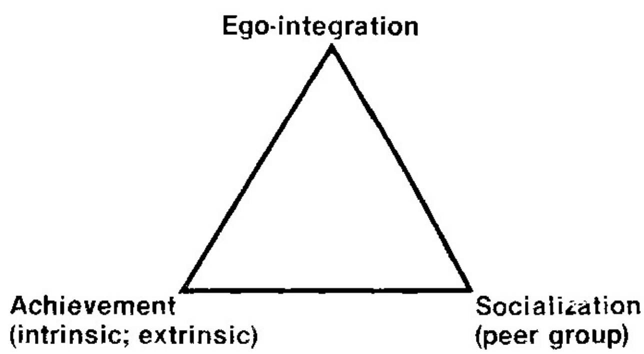

Figure 3. Model of Theories of Motivation

An important characteristic of motivation, whatever its source(s), is attention. The ability to focus attention on a cognitive task is closely related to the amount and quality of learning. Neuropsychological research implicates the brain stem as being critical to focusing of attention and consciousness. And more specifically, it seems to be a particular structure with the brain stem -- the reticular formation, or the reticular system. Putatively, all neural messages that flow between the sensory receptors and the brain, and between the brain and the effectors (glands and muscles) are registered in the reticular system. The messages are also regulated and modified here. In a way, the reticular system can be likened to the set of toll booths at a particular location on a superhighway.

All of us have at times worked with children and youth who have been diagnosed as hyperkinetic or learning disabled. They are unable to focus attention on cognitive tasks. They are highly distractable. There is good reason to believe that a malfunctioning of the reticular system prevents it from screening inappropriate messages coming in from the sensory receptors. The malfunction is caused more likely by a biochemical disturbance than by brain stem pathology. Quite often these children who are unable to attend to school-type tasks respond favorably to appropriate medication.

An educationally useful level of motivation seems to be midway on a continuum. At one end is apathy which can be induced by neurological, physiological (i.e. undernourishment), psychological, or educational causes. At the other extreme is wild, uncontrolled excitement. We have all seen this in some of the demonstration lessons that were popular in the days of the old new math. Neuropsychologists are presently studying the relationship between attention and various areas of the brain. We can look forward to receiving useful information in the years immediately ahead.

There are other primary sources for an instructional psychology of mathematics for which knowledge in neuropsychology has significant implications. Not the least of these is theories of human development. And of these, it seems that there are relationships between the stages of Piagetian development and growth spurts in the development of the brain. We can expect new knowledge in this area to contribute to our expanding and increasingly more professional view of our work in diagnostic-prescriptive teaching of mathematics.

Concluding Statement - Towards a Systems Theory

In concluding these remarks I would like to draw your attention to the emerging work in systems theory in medicine. It may have implications for our work as we seek some guiding principles for the years ahead. Our major task is to continue to search the literature of other disciplines from A to Z, anthropology to zoology, for proven ideas that can be integrated into our emerging instructional psychology of mathematics and thereby create a more functional system.

Systems theory in medicine is concerned on the one hand with the interrelationships among the basic disciplines from which the practice of medicine draws its principles. On the other hand it is concerned with Miller's (1978) seven levels of complexity, simple and single to multiple and complex. In medicine one can ask how "the will to live" translates into longer life. Exactly what are the interrelationships between the cell level of "will to live" and the organism level of better health. Ultimately the psychological action "will to live" is chemically mediated at the cellular level. Exactly what happens in the cells, which is translated to body organs, which is further translated to the individual, and on to higher levels, the family, the society, etc.?

In the same way, the "will to succeed" or the "will to fail" in mathematics is surely more complex behavior than what we see on the child's paper. As professional people, it behooves us to keep ourselves informed on the developments in other disciplines in these years when so much new knowledge is being created, as in neuropsychology. It is imperative if we wish to further develop the instructional psychology of mathematics from which we draw our principles and methods for the diagnostic and prescriptive teaching of mathematics.

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